Chemistry 123 **Problem Set Solution** **Winter Term 2003**
Assignment #1 **Instructor: R. Rossi**

Common Symbols Rob uses in this key, or more generally when writing stuff on the board in class:

m = mass

m_x = mass of x, for example, m_{cheese} would represent the mass of cheese present

η = moles

η_x = moles of x, for example, η_{beans} would represent the moles of beans present

$\hat{X}$ = The extensive property X measured on a mass basis, for example, an energy in joules per gram would be $\hat{E}$

$\tilde{X}$ = The extensive property X measured on a mole basis, for example, an energy in joules per mole would be $\tilde{E}$

$\widehat{X}$ = The extensive property X measured on a volume basis, for example, an energy in joules per liter would be $\widehat{E}$

$\tilde{m}_x$ = molar mass of substance x

sf = significant figures

$\%_{mass}$ = percent by mass

$\%_{mol}$ = mole percent = percent by number

$\%_{vol}$ = percent by volume

T = temperature

P = pressure

π = osmotic pressure

V = volume

ℓ = liters (litres)

R = universal gas constant

ρ = density

g = the acceleration of gravity at sea level on earth

F = force

a = acceleration

A = area

sat'd = saturated

sol'n = solution

P_x = partial pressure of gas x

P^o_x = pure component vapor pressure of x

m_x = molality of substance x

M = molarity

[A] = molarity of chemical species A

atm = atmosphere(s)

$\underline{x}$ = an exact number For example, 1000 would be understood to have 1 significant figure. $\underline{1000}$ is understood to be *exact*

I employ the standard abbreviations for SI units and prefixes, which appear in Tables 1.1 and 1.2 in Zumdahl.

1. Aerozine has been displaced by other fuels because it is environmentally expensive; during the space race, however, little attention was paid to the possible environmental costs of the space program.

   a. I'm going to do this using my moron-proof balancing method – I have to do this, because I am a moron. For the rest of the world, this is a good way to avoid making simple mistakes. Geniuses can balance reactions of this complexity "by inspection," so no work needs to be shown, but I recommend this approach to anyone who has any trouble balancing reactions in other ways. Keep adding *molecules* and crossing out *atoms* until everything is gone. Then add up the number of each type of molecule you had to use in order to complete the process, and that is your balanced chemical reaction:

$$\begin{array}{ccccccc}
\cancel{N}_2\cancel{H}_4 & + & \cancel{N}_2\cancel{O}_4 & \rightarrow & \cancel{N}_2 & + & \cancel{H}_2\cancel{O} \\
\cancel{N}_2\cancel{H}_4 & & & & \cancel{N}_2 & & \cancel{H}_2\cancel{O} \\
 & & & & \cancel{N}_2 & & \cancel{H}_2\cancel{O} \\
 & & & & & & \cancel{H}_2\cancel{O} \\
\hline
2\,N_2H_4 & + & N_2O_4 & \rightarrow & 3\,N_2 & + & 4\,H_2O
\end{array}$$

   The same process works for the second reaction, though you have to be careful to realize that a single "$(CH_3)_2$" unit contains a total of two carbon atoms and six hydrogen atoms:

$$\begin{array}{ccccccccc}
(\cancel{C}\cancel{H}_3)_2\cancel{N}_2\cancel{H}_2 & + & \cancel{N}_2\cancel{O}_4 & \rightarrow & \cancel{N}_2 & + & \cancel{H}_2\cancel{O} & + & \cancel{C}\cancel{O}_2 \\
 & & \cancel{N}_2\cancel{O}_4 & & \cancel{N}_2 & & \cancel{H}_2\cancel{O} & & \cancel{C}\cancel{O}_2 \\
 & & & & \cancel{N}_2 & & \cancel{H}_2\cancel{O} & & \\
 & & & & & & \cancel{H}_2\cancel{O} & & \\
\hline
(CH_3)_2N_2H_2 & + & 2\,N_2O_4 & \rightarrow & 3\,N_2 & + & 4\,H_2O & + & 2\,CO_2
\end{array}$$

   So the balanced reactions are as follows:

$$\boxed{\begin{array}{c}
2\,N_2H_4 + N_2O_4 \rightarrow 3\,N_2 + 4\,H_2O \\
(CH_3)_2N_2H_2 + 2\,N_2O_4 \rightarrow 3\,N_2 + 4\,H_2O + 2\,CO_2
\end{array}}$$

   b. Now that we have the balanced reactions, we can calculate how much N_2O_4 would be needed to react completely with the given 1420 kg of Aerozine. Since Aerozine is 50.0% hydrazine by mass, that means there we could expect to find 0.500 kg of hydrazine in exactly one kilogram of Aerozine 50. Using dimensional analysis, we find that there are

$$1420\ \text{kg Aerozine} \times \left(\frac{0.500\ \text{kg}\ N_2H_4}{\underline{1}\ \text{kg Aerozine}} \right) = 710.\ \text{kg}\ N_2H_4$$

   in 1420 kg of Aerozine 50. Note that this amount is not written as 710 kg, but rather as 710. kg, with the trailing decimal point. That signifies that the trailing zero is a significant figure, and that we know the actual number of kilograms of N_2H_4 was between 709 and 711. This is, in turn, a result of the fact that the two values multiplied together, 1420 kg and 50.0%, each have three significant figures. If this is puzzling, read §1.5 in Zumdahl, it is important!
   It's important to remember that the other 50.0% of the fuel is dimethylhydrazine, that is, we also have

$$1420\ \text{kg Aerozine} \times \left(\frac{0.500\ \text{kg}\ (CH_3)_2N_2H_2}{\underline{1}\ \text{kg Aerozine}} \right) = 710.\ \text{kg}\ (CH_3)_2N_2H_2$$

   So, we are in effect asked to determine how much N_2O_4 is needed to effect the complete combustion of 710. kg of hydrazine AND 710. kg of dimethylhydrazine. We will want to calculate the molar masses of the compounds involved, and do so to at least three significant figures because we don't want the molar mass value to limit the accuracy of our result in any case where we can avoid it:

$$\tilde{m}_{N_2H_4} = \underline{2}(\tilde{m}_N) + \underline{4}(\tilde{m}_H) = \underline{2}\left(14.01\frac{g}{mol}\right) + \underline{4}\left(1.008\frac{g}{mol}\right) = 28.02\frac{g}{mol} + 4.032\frac{g}{mol} = 32.05_2\ \frac{g}{mol}$$

$$\begin{aligned}
\tilde{m}_{(CH_3)_2N_2H_2} &= \underline{2}(\tilde{m}_{CH_3}) + \underline{2}(\tilde{m}_N) + \underline{2}(\tilde{m}_H) = \underline{2}\left[\tilde{m}_C + \underline{3}(\tilde{m}_H)\right] + \underline{2}(\tilde{m}_N) + \underline{2}(\tilde{m}_H) \\
&= \underline{2}(\tilde{m}_C) + \underline{6}(\tilde{m}_H) + \underline{2}(\tilde{m}_N) + \underline{2}(\tilde{m}_H) = \underline{2}(\tilde{m}_C) + \underline{2}(\tilde{m}_N) + \underline{8}(\tilde{m}_H) \\
&= \underline{2}\left(12.01\frac{g}{mol}\right) + \underline{2}\left(14.01\frac{g}{mol}\right) + \underline{8}\left(1.008\frac{g}{mol}\right) = 24.02\frac{g}{mol} + 28.02\frac{g}{mol} + 8.064\frac{g}{mol} = 60.10_4\ \frac{g}{mol}
\end{aligned}$$

$$\tilde{m}_{N_2O_4} = \underline{2}(\tilde{m}_N) + \underline{4}(\tilde{m}_O) = \underline{2}\left(14.01\frac{g}{mol}\right) + \underline{4}\left(16.00\frac{g}{mol}\right) = 28.02\frac{g}{mol} + 64.00\frac{g}{mol} = 92.02\frac{g}{mol}$$

Problem 1(b), continued

Dimensional analysis carries us through the rest of the calculations, using the molar masses and stoichiometric coefficients we have calculated up to this point. Note that I am using kilomoles, which is nothing magical. I can stick any prefix I want onto a conversion factor, as long as I do it for both the numerator and denominator. Thus if

$$\left(\frac{32.05_2\ \text{g}\ N_2H_4}{\text{mol}\ N_2H_4}\right) \text{ is a valid conversion factor, then so is } \left(\frac{32.05_2\ \text{kg}\ N_2H_4}{\text{kmol}\ N_2H_4}\right)$$

because a kilomole is 1000 moles, just as a kilogram is 1000 grams. Ok, so complete combustion of the N_2H_4 will require

$$710.\ \text{kg}\ N_2H_4\left(\frac{\underline{1}\ \text{kmol}\ N_2H_4}{32.05_2\ \text{kg}\ N_2H_4}\right)\left(\frac{\underline{1}\ N_2O_4}{\underline{2}\ N_2H_4}\right)\left(\frac{92.02\ \text{kg}\ N_2O_4}{\underline{1}\ \text{kmol}\ N_2O_4}\right) = 101_{9.19}\ \text{kg}\ N_2O_4$$

while complete combustion of the $(CH_3)_2N_2H_2$ will require

$$710.\ \text{kg}\ (CH_3)_2N_2H_2\left(\frac{\underline{1}\ \text{kmol}\ (CH_3)_2N_2H_2}{60.10_4\ \text{kg}\ (CH_3)_2N_2H_2}\right)\left(\frac{\underline{2}\ N_2O_4}{\underline{1}\ (CH_3)_2N_2H_2}\right)\left(\frac{92.02\ \text{kg}\ N_2O_4}{\underline{1}\ \text{kmol}\ N_2O_4}\right) = 217_{4.04}\ \text{kg}\ N_2O_4$$

Thus the total mass of N_2O_4 required for the ascent will be ($101_{9.19}$ + $217_{4.04}$ =) $319_{3.23}$ kg N_2O_4. Note that I have kept track of significant figures without rounding off. I could just report this number with the subscripted insignificant digits, but if I want to round off, now would be the time: right before reporting the answer.

The ascent will require $319_{3.23}$ kg of N_2O_4, or 3190 kg of N_2O_4 (give or take 10 kg)

c. Since only $(CH_3)_2N_2H_2$ leads to the production of CO_2, in this part we need only account for the 710. kg of it present in the fuel mixture. However, we will also need to calculate the molar mass of carbon dioxide:

$$\tilde{m}_{CO_2} = \tilde{m}_C + \underline{2}(\tilde{m}_O) = 12.01\frac{\text{g}}{\text{mol}} + \underline{2}\left(16.00\frac{\text{g}}{\text{mol}}\right) = 12.01\frac{\text{g}}{\text{mol}} + 32.00\frac{\text{g}}{\text{mol}} = 44.01\frac{\text{g}}{\text{mol}}$$

$$710.\ \text{kg}\ (CH_3)_2N_2H_2\left(\frac{\underline{1}\ \text{kmol}\ (CH_3)_2N_2H_2}{60.10_4\ \text{kg}\ (CH_3)_2N_2H_2}\right)\left(\frac{\underline{2}\ CO_2}{\underline{1}\ (CH_3)_2N_2H_2}\right)\left(\frac{44.01\ \text{kg}\ N_2O_4}{\underline{1}\ \text{kmol}\ N_2O_4}\right) = 103_{9.77}\ \text{kg}\ N_2O_4$$

The ascent will generate 1040 kg of CO_2 as a result of the combustion of the dimethylhydrazine in the fuel.

d. Both fuel components generate water when burned, so this part will resemble (b). We'll need the molar mass of water:

$$\tilde{m}_{H_2O} = \underline{2}(\tilde{m}_H) + \tilde{m}_O = \underline{2}\left(1.008\frac{\text{g}}{\text{mol}}\right) + 16.00\frac{\text{g}}{\text{mol}} = 2.016\frac{\text{g}}{\text{mol}} + 16.00\frac{\text{g}}{\text{mol}} = 18.01_6\frac{\text{g}}{\text{mol}}$$

The complete combustion of the hydrazine in the fuel will generate

$$710.\ \text{kg}\ N_2H_4\left(\frac{\underline{1}\ \text{kmol}\ N_2H_4}{32.05_2\ \text{kg}\ N_2H_4}\right)\left(\frac{\underline{4}\ H_2O}{\underline{2}\ N_2H_4}\right)\left(\frac{18.01_6\ \text{kg}\ H_2O}{\underline{1}\ \text{kmol}\ H_2O}\right) = 798._{16}\ \text{kg}\ H_2O$$

while complete combustion of the $(CH_3)_2N_2H_2$ will generate

$$710.\ \text{kg}\ (CH_3)_2N_2H_2\left(\frac{\underline{1}\ \text{kmol}\ (CH_3)_2N_2H_2}{60.10_4\ \text{kg}\ (CH_3)_2N_2H_2}\right)\left(\frac{\underline{4}\ H_2O}{\underline{1}\ (CH_3)_2N_2H_2}\right)\left(\frac{18.01_6\ \text{kg}\ H_2O}{\underline{1}\ \text{kmol}\ H_2O}\right) = 851._{28}\ \text{kg}\ H_2O$$

Thus we can calculate the total mass of water produced, and using the density of water given in the problem, get our answer:

Total mass of water = $798._{16}$ + $851._{28}$ = $1649._{44}$ kg of H_2O (Be sure you understand why this has four sf!)

$$1649._{44}\ \text{kg}\ H_2O\left(\frac{1.00\ \text{m}\ell\ H_2O}{\underline{1}\ \text{g}\ H_2O}\right)\left(\frac{\underline{1000}\ \text{g}}{\underline{1}\ \text{kg}}\right)\left(\frac{\underline{1}\ \ell}{\underline{1000}\ \text{m}\ell}\right) = 164_{9.44}\ \ell\ \text{of}\ H_2O$$

Note that the number of significant figures has dropped back down to three again, because of the density.

1650 ℓ of water were generated in the ascent

2 a. There are plenty of hints in the problem intended to point us toward treating this as an ideal gas problem. But this is a "real life" ideal gas problem, and dealing with the vagaries of real life has tripped up many the aspiring ideal gas law expert! It's one thing to plug numbers into an equation, quite another to know how to choose the right numbers to plug in. Let me re-iterate the note in the assignment: the ideal gas law requires *absolute* quantities, that is, quantities measured on a scale wherein zero is the lowest one can possibly go. In our everyday world, neither temperature nor pressure are often measured on such a scale. So you have to be savvy about converting such measurements into absolute units!

Another aspect of this problem to which you may not have been previously exposed is the limited information: you aren't given the volume of the tire, or the number of moles of gas inside it. That leaves you one of three options, namely:

1. Use the ideal gas law, assuming a volume for the tires or a number of moles of gas inside them. This is called choosing a "basis," and it works when it doesn't matter what the value of your assumed value is, so long as it is constant.

2. Use the ideal gas law, but manipulate it so that the things you don't know, the volume and moles of gas, drop out.

3. Use the relation P/T = a constant, which applies to a fixed volume and quantity of gas (the dead white guy who gets his name slapped on this one is Joseph-Louis Gay-Lussac, although he actually discovered Charles' Law…it's wack.)

These all amount to the same thing, but for most people one or the other is quite comfortable while the others are disconcerting. You would do well to get over any such hang-ups at the first opportunity! Think carefully about how these will lead to the same result. In working this problem out I will adopt approach (2), because I will use (1) in the next problem and because (3) is not generally an option; few general relationships have developed so gradually and their history so lovingly remembered as the ideal gas law, such that every possible derived relationship is not only detailed in books, but has its own name!

OK, so in order to adopt any of these approaches, we'll have to assume that the volume of my tires is constant. That may not be rigorously true, but it is certainly very close to the truth. Tires don't expand much once they are full! It's not like you hook up an air pump and your tires go from pizza-cutters to dune buggy with a couple injections of air. Tires don't swell to an appreciable multiple of their original volume, particularly not the steel-belted radials of today. OK, what about assuming that the number of moles of gas is constant? Well, that basically means we are assuming that the tires don't leak. Actually, all tires leak at some non-zero rate, but the fraction of the gas in tire likely to leak out over the course of a single day better not be much! If it is, i.e. if someone knifes my tire, the problem really isn't meaningful anymore. Does all this mean that you have to treat the tires as having a constant volume and containing a constant quantity of gas? Nope, you could certainly assume something else, say that the volume changes by 1%, and that the moles of gas present in each tire drops by 0.5%. But it certainly makes the problem harder, and it's not likely to get you an answer appreciably more accurate than that we'll get here assuming both are constant. The take-home lesson is **assume the simplest things you can that don't make a problem unrealistic!**

The ideal gas law says that $PV = \eta RT$ or, put another way, $\frac{PV}{\eta T} = R = \text{a constant}$. In this particular case, we've also assumed η and V to be constant, and so we expect $\frac{P}{T} = \frac{R\eta}{V} = \text{a constant}$. So if we look at this system under two different conditions, call them the initial condition *i* and the final condition *f*, we can expect that $\frac{P_i}{T_i} = \frac{R\eta}{V} = \frac{P_f}{T_f} \Rightarrow \frac{P_i}{T_i} = \frac{P_f}{T_f}$. So if we know the initial and final temperature in the tires, and the initial pressure, we can calculate the final pressure. Hmmm, so what's all that extraneous extra data in the problem then, like the pressure up at Henry's Lake?!? With me it's not far-fetched to think it's exactly that: extraneous extra data. Life is full of it, and so are my problems. But hey, in this case it turns out a lot of it isn't so extraneous, because in order to use this equation, a derivative of the ideal gas law, we need to use *absolute* temperature and pressure. The tire pressure readings given in the problem are *gauge* pressures, that is, they are measurements of the difference in pressure inside the tire vs. outside of it. The temperature readings are *relative* temperatures, that is, they are measured relative to the temperature of something else…in the case of the Fahrenheit scale, it turns out to be the freezing point of salty water. So let's translate each value into absolute units, shall we? First, the temperatures, since that is probably most familiar to you. The initial temperature of my tires was probably that of the air air in Northfield, 76ºF. (That's another assumption!) To get this onto an absolute scale, we could either adjust for the value of absolute zero on the Fahrenheit (that's -459.67ºF, and after the adjustment you end up on what's called the Rankine scale…engineers in the U.S. use it all the time), or convert to the Celsius scale and then adjust for the value of absolute zero on the Celsius scale (-273.15ºC) to get to Kelvins. Doing the latter, and calling the time of our departure from Northfield the initial condition, and the morning in Idaho a few days later the final condition:

$$\text{Initial temperature} = T_i = 76^\circ\text{F} = \frac{5}{9}(76-\underline{32})^\circ\text{C} = \frac{5}{9}(44)^\circ\text{C} = 24._{444}{}^\circ\text{C} = (24._{444} + 273.15)\text{ K} = 297._{594}\text{ K}$$

$$\text{Final temperature} = T_i = 42^\circ\text{F} = \frac{5}{9}(42-\underline{32})^\circ\text{C} = \frac{5}{9}(10.)^\circ\text{C} = 5.5_{56}{}^\circ\text{C} = (5.5_{56} + 273.15)\text{ K} = 278.7_{06}\text{ K}$$

Problem 2(a), continued…

Now, I measured the "tire pressure" to be 32 psig in Northfield. That means that the pressure inside the tire was 32 psi higher than the pressure outside the tire, viz., the pressure of the atmosphere. To get the absolute pressure inside the tire at the start of the problem, we need to convert the two pressures to the same units and then add them together, just as we did with the temperature. We can work in whatever units we like, but since we have the gas constant in terms of atmospheres, it's probably easiest to change both of the given pressure values into those units. Besides, atmospheres are a nice, intuitive unit that give you a good idea how much pressure we're really talking about. Okie dokie, here goes nothing:

$$P_{\text{tire}} = 32 \text{ psi gauge}\left(\frac{\underline{1} \text{ atmosphere}}{14.70 \text{ psi}}\right) = 2.1_{77} \text{ atmospheres gauge pressure}$$

$$P_{\text{atm}} = 29.0 \text{ in Hg}\left(\frac{\underline{\underline{2.54}} \text{ cm}}{\underline{1} \text{ inch}}\right)\left(\frac{\underline{10} \text{ mm}}{\underline{1} \text{ cm}}\right)\left(\frac{\underline{1} \text{ atmosphere}}{\underline{760} \text{ mm Hg}}\right) = 0.969_2 \text{ atmospheres atmospheric pressure}$$

Initial pressure = $P_i = P_{\text{tire}}$ = gauge pressure + atmospheric pressure = 2.1_{77} atm + 0.969_2 atm = 3.1_{462} atm absolute

We can now employ our simplified ideal gas law formula and calculate what the final pressure ought to be, provided our assumptions hold water, that is, assuming my tires don't leak much, don't change volume much, and reach thermal equilibrium with the air surrounding them by the time the sun rises (before the sun starts heating them up!):

$$\frac{P_i}{T_i} = \frac{P_f}{T_f} \quad \Rightarrow \quad P_f = T_f \frac{P_i}{T_i} = (278.7_{06} \text{ K})\frac{(3.1_{462} \text{ atma})}{(297._{594} \text{ K})} = 2.9_{465} \text{ atm absolute}$$

To figure out what my tire pressure gauge would have read, we subtract off the atmospheric pressure present when I took the measurement (23.7 in Hg) to find the difference between the pressure inside the tire and that outside the tire, the gauge pressure:

$$P_f = 2.9_{465} \text{ atma} - 23.7 \text{ in Hg}\left(\frac{\underline{\underline{2.54}} \text{ cm}}{\underline{1} \text{ inch}}\right)\left(\frac{\underline{10} \text{ mm}}{\underline{1} \text{ cm}}\right)\left(\frac{1 \text{ atm}}{\underline{760} \text{ mm Hg}}\right) = 2.9_{465} \text{ atma} - 0.792_{08} \text{ atm} = 2.1_{544} \text{ atm gauge pressure (atmg)}$$

Since the tire pressure gauge reads in psi rather than atm, we need to do one more simple conversion:

$$P_f = 2.1_{544} \text{ atmg}\left(\frac{14.70 \text{ psi}}{\underline{1} \text{ atm}}\right) = 31._{67} \text{ psi gauge pressure (psig)}$$

Well, golly. This isn't a measureable change for my tire gauge, which is only good to ± 1 psig. Note that if the temperature alone had been taken into account, the gauge pressure would have gone down by 2.9 psi, and if the change in atmospheric pressure were the only operating factor, the gauge pressure would have gone up by 2.6 psi. As it actually is, the two factors nearly balance each other out. While we know the pressure has indeed changed, we also know that I don't consider my pressure gauge reliable to any better than the nearest psi…and so I won't be able to tell that anything has changed:

The pressure on my tire gauge should read 32 psig when I check my tires at Henry's Lake.

2 b. Modern snack chip bags are sealed systems, and are actually filled with nitrogen at the factory to keep the chips fresh and to help keep them from getting crushed into dust. (That's not critical to solving this problem, I just though you might find it interesting; what is critical is that they contain a fixed amount of gas.) Since I kept the temperature in my car reasonably constant, and the bags were full of gas but the bags can't really expand like a balloon can, we have a fixed amount of gas at essentially constant temperature and volume. That means that the *absolute* pressure of the gas in the bags remains roughly constant, which you may not have needed any convincing about, but if you did, there it was. If the pressure outside the bag is equal to that inside the bag, the bag just sits there. If the pressure outside is higher than that inside, the bag crumples up. (If you buy chips in Colorado and then drive down to Death Valley, this is what you'll experience!) If the pressure inside the bag is higher than that outside, the bag swells up bigger (or at least it tries to). The difference between the pressure outside the bag (the atmospheric pressure) and that inside the bag is the *gauge* pressure. So in other words, the condition of the bag, normal, "flat," or "fat," depends on the gauge pressure. The pressure inside the bag stays reasonably constant, but as I drive up into the mountains, the atmospheric pressure steadily drops. As a result the gauge pressure gets ever larger, causing the bag first to puff up like the Sta-Puff marshmallow man, stretching and shuffling as it pushes other things around and the plastic strains under the pressure, and then to finally pop when a seam on the bag gives way and the gas trapped inside escapes.

For you mathematical types, $P_{\text{gauge}} = P_{\text{absolute}} - P_{\text{atmosphere}}$ = constant – (decreasing) = an increasing quantity ⇒ a swelling bag!

This really did happen to my girlfriend and I! Thankfully, we were able to figure out what happened so it didn't freak us out!

3. If you don't really understand concentration and concentration units, this will be a tough problem! The two questions are phrased to try to make the problem easier for you. Calculating the molarity of the NaOH solution in (b) is downright tricky. (See below.)

   a. The easy way to do this part is to choose a convenient "basis" for your calculations. Solubility is an intensive property, meaning that the solubility of sodium hydroxide in water doesn't depend on the amount of water you have. Sure, the amount of NaOH you need to pour into a vat of water in order to saturate it does increase with the amount of water in there, but solubilities are given in the form (amount of solute/amount of solvent) or something akin to it, and since the numerator goes up with the denominator, the actual ratio stays constant. So if we can calculate the solubility of NaOH in any given amount of water, we'll have it for any other amount of water. Let's examine the given information for clues as to what might be a good basis. We are told that the solubility of NaOH in H_2O is 20.378 moles of NaOH per liter of *solution* (M). So if we have one liter of saturated sodium hydroxide solution, we know that it contains 20.378 moles of NaOH. We are also told that the density of saturated NaOH in H_2O is 1.53 g/mℓ: hence one milliliter of saturated NaOH solution will have a mass of 1.53 grams. Convenient bases for this problem might therefore be one milliliter, or one liter, of saturated NaOH solution. I'm going to go with a whole liter in this case.

   As I just explained, one liter of saturated NaOH solution will contain 20.378 moles of NaOH. But how much water is in there? It is critical to realize that it's *not one liter!* The liter of solution we have chosen as our basis includes a sizable volume occupied by sodium and hydroxide ions. The volume of water present in our sample is less than one liter.

   It is important to realize that volume is not strictly conserved when you form a solution. For example, if you mix 100. mℓ of ethanol with 100. mℓ of water, you end with about 185 mℓ of solution. On the other hand, *mass is strictly conserved in any chemical reaction*, including a dissolution reaction like the one we are considering here,

$$\text{NaOH}_{(s)} \xrightarrow{\text{water}} \text{Na}^+_{(aq)} + \text{OH}^-_{(aq)}$$

   If we mix 100. g of water with 100. g of ethanol, we can be sure we'll end up with 200. g of solution. Not only that, we will still have 100. g of water in that solution, and 100. g of ethanol in it. This is called **conservation of mass**, and it is going to help us big time in this problem. How so? Well, first, since we know the density of our saturated NaOH solution, we can readily determine the mass of our $\underline{1}$ ℓ sample of the stuff:

$$\underline{1}\ \ell \text{ sat'd NaOH}_{(aq)} \times \left(\frac{1.53 \text{ g sat'd NaOH}_{(aq)}}{\underline{1} \text{ m}\ell \text{ sat'd NaOH}_{(aq)}}\right)\left(\frac{\underline{1000} \text{ m}\ell}{\underline{1} \text{ m}\ell}\right) = 153_0 \text{ g sat'd NaOH}_{(aq)}$$

   We also know that this sample contains 20.278 moles of NaOH, and since we can calculate the molar mass of NaOH, we can determine the mass of NaOH in our sample:

$$\tilde{m}_{\text{NaOH}} = \tilde{m}_{\text{Na}} + \tilde{m}_{\text{O}} + \tilde{m}_{\text{H}} = 22.99\frac{\text{g}}{\text{mol}} + 16.00\frac{\text{g}}{\text{mol}} + 1.008\frac{\text{g}}{\text{mol}} = 39.99_8 \frac{\text{g NaOH}}{\text{mol NaOH}}$$

$$20.378 \text{ mol NaOH} \times \left(\frac{39.99_8 \text{ g NaOH}}{\underline{1} \text{ mol NaOH}}\right) = 815.0_{79} \text{ g NaOH}$$

   Now we can use the law of conservation of mass to figure out the mass of the water in this solution:

$$m_{\text{solution}} = m_{\text{NaOH}} + m_{H_2O} \quad\Rightarrow\quad 153_0 \text{ g} = 815.0_{79} \text{ g} + m_{H_2O} \quad\Rightarrow\quad m_{H_2O} = 153_0 \text{ g} - 815.0_{79} \text{ g} = 71_{4.921} \text{ g}$$

   Believe it or not, that's it! We now know the mass of NaOH present in a known mass of water composing a saturated solution of sodium hydroxide. We just need to express it in the requested units:

$$\text{Solubility of NaOH in } H_2O = \left(\frac{815.0_{79} \text{ g NaOH}}{71_{4.921} \text{ g } H_2O}\right) = \left(\frac{? \text{ g NaOH}}{\underline{100} \text{ g } H_2O}\right)$$

$$\Rightarrow\ ? \text{ g NaOH} = \underline{100} \text{ g } H_2O \times \left(\frac{815.0_{79} \text{ g NaOH}}{71_{4.921} \text{ g } H_2O}\right) = 11_{4.01} \text{ g of NaOH per 100 g of water}$$

The solubility of NaOH in H_2O at room temperature is 11_4 g of NaOH per 100 g of H_2O.

   It is interesting to note that this solution is so concentrated that it is over 50% by mass solute! That's a lot of drain opener!

b. This part of the problem is actually much more straightforward. It's simply a matter of archaic unit conversions to go from the given information to the requested concentration:

$$1 \text{ cup solid NaOH} \times \left(\frac{\underline{1} \text{ qt}}{\underline{4} \text{ cups}}\right)\times\left(\frac{\underline{1000} \text{ m}\ell}{1056.68 \text{ qt}}\right)\times\left(\frac{\underline{1000} \text{ m}\ell}{\underline{1}\ \ell}\right) = 236.6 \text{ m}\ell \text{ solid NaOH}$$

$$236.6 \text{ m}\ell \text{ solid NaOH} \times \left(\frac{2.13 \text{ g NaOH}}{\underline{1} \text{ m}\ell \text{ solid NaOH}}\right) = 503._9 \text{ g solid NaOH}$$

To figure out what the mass of a gallon of water is, we are going to need the density of water. Hmmm, you say, that's not given in the problem. Nope, it's not, not even in a sneaky way, not even in the hints. What's up with that?!? Just because something isn't given in a problem doesn't mean it ceases to exist. You can quite readily find the density of water at room temperature…in fact, it even has its own heading in the index of your text. You could get it from some other place, and you could get it with a different number of significant figures, but you can get it. (My number is from Table 1.5 of Zumdahl, on p. 27.) The take-home lesson is that I will not always include every number you need in a problem description! In some cases you may have to look things up! Ok, back to our gallon of water:

$$1 \text{ gallon } H_2O \times\left(\frac{\underline{1000}\ \ell}{264.17 \text{ gal}}\right)\times\left(\frac{\underline{1000} \text{ m}\ell}{\underline{1}\ \ell}\right)\times\left(\frac{0.998 \text{ g } H_2O}{\underline{1} \text{ m}\ell\ H_2O}\right) = 377_8 \text{ g } H_2O$$

So now we know that the Gunk-Out solution will, giving them all the benefit of the doubt we possibly can, be only

$$\frac{503._9 \text{ g of NaOH}}{377_8 \text{ g of } H_2O} = \frac{?\ \text{g of NaOH}}{\underline{100} \text{ g of } H_2O} \quad\Rightarrow\quad ?\ \text{g of NaOH} = \underline{100} \text{ g of } H_2O\left(\frac{503._9 \text{ g of NaOH}}{377_8 \text{ g of } H_2O}\right) = 13.3_{38}$$

The tub of properly-prepared Gunk Out will contain, at best, 13.3_{38} g of NaOH per 100 g of water,and it will be far from saturated. There is no way that Gunk-Out's claim is valid, and it fails the "Commercial Challenge!"

As David Horowitz used to say, "Stay aware and informed, and don't let *anyone* rip you off!" Er, something like that.

4. The math in this problem is not hard. The hard part is figuring out what math to do. It will be very helpful to understand that the percent by "x" of a substance "A" is defined generally as

$$\%_x \text{ of A} \equiv \frac{\text{x of A}}{\text{total x}} \qquad \text{for example, if x = mass, } \%_{mass} \text{ of A} \equiv \frac{\text{mass of A}}{\text{total mass}}$$

The average of something is slightly different, it is defined as

$$\text{Average x of A} \equiv \frac{\text{total x of all A's in sample}}{\text{number of A's in sample}}$$

$$\text{for example, if x = mass, then the average mass of A} \equiv \frac{\text{toal mass of all A's in sample}}{\text{number of A's in sample}}$$

a. To determine the average atomic mass of naturally-occurring Ne atoms, it will be easiest if we visualize a concrete sample of such atoms to think about. A convenient number is 100,000 Ne atoms. If we have 100,000 Ne atoms,

90.92% of the atoms will be ^{20}Ne →

$$\frac{0.9092\ {}^{20}\text{Ne atoms}}{1 \text{ naturally-occuring Ne atom}} \times 100{,}000 \text{ naturally-occuring Ne atoms} = 90920\ {}^{20}\text{Ne atoms}$$

0.257% of the atoms will be ^{21}Ne →

$$\frac{0.00257\ {}^{21}\text{Ne atoms}}{1 \text{ naturally-occuring Ne atom}} \times 100{,}000 \text{ naturally-occuring Ne atoms} = 257\ {}^{21}\text{Ne atoms}$$

8.82% of the atoms will be ^{22}Ne →

$$\frac{0.0882\ {}^{22}\text{Ne atoms}}{1 \text{ naturally-occuring Ne atom}} \times 100{,}000 \text{ naturally-occuring Ne atoms} = 8820\ {}^{22}\text{Ne atoms}$$

Problem 4(a), continued…

The total mass of all the atoms will be

$$\left(90920\ {}^{20}\text{Ne atoms} \times \frac{20\ \text{a.m.u}}{1\ {}^{20}\text{Ne atom}}\right)+\left(257\ {}^{21}\text{Ne atoms} \times \frac{21\ \text{a.m.u}}{1\ {}^{21}\text{Ne atom}}\right)+\left(8820\ {}^{22}\text{Ne atoms} \times \frac{22\ \text{a.m.u}}{1\ {}^{22}\text{Ne atom}}\right)$$

$$= 1818_{400}\ \text{amu} + 539_{7}\ \text{amu} + 194_{040}\ \text{amu} = 2017_{837}\ \text{amu} = 2018000\ \text{amu}$$

and so the average mass of the atoms in this 100,000 atom sample will be

$$\text{average mass of Ne} = \frac{\text{toal mass of all Ne atoms in sample}}{\text{number of Ne atoms in sample}} = \frac{2018000\ \text{amu}}{100000\ \text{atoms}} = 20.18\ \text{amu average}$$

Whaddaya know, that's the value in the front of your book. Ain't internal consistency grand?

b. Let's again use a sample of 100,000 Ne atoms. We have calculated all the numbers we need in part (a):

$$\%_{\text{mass}}\ \text{of}\ {}^{20}\text{Ne} = \frac{\text{mass of}\ {}^{20}\text{Ne}}{\text{total mass of Ne}} = \frac{1818_{400}\ \text{amu}}{2017_{837}\ \text{amu}} = 0.9011_{63} \times 100\% = 90.12\%$$

$$\%_{\text{mass}}\ \text{of}\ {}^{21}\text{Ne} = \frac{\text{mass of}\ {}^{21}\text{Ne}}{\text{total mass of Ne}} = \frac{539_{7}\ \text{amu}}{2017_{837}\ \text{amu}} = 0.00267_{46} \times 100\% = 0.267\%$$

$$\%_{\text{mass}}\ \text{of}\ {}^{22}\text{Ne} = \frac{\text{mass of}\ {}^{22}\text{Ne}}{\text{total mass of Ne}} = \frac{194_{040}\ \text{amu}}{2017_{837}\ \text{amu}} = 0.0961_{624} \times 100\% = 9.62\%$$

It's both wise and reassuring to check that 90.12% + 0.267% + 9.62% = 100.01%, which is 100.00% within its implicit uncertainty of ±.01%. Note that the mass percentages are only slightly different from the atomic percentages, because the masses of ^{20}Ne, ^{21}Ne, and ^{22}Ne are very similar. The heaviest isotope, ^{22}Ne makes up a larger fraction of the mass than it does of the atoms, which makes sense if you think about it a little bit.

c. You don't have to do any calculations at all in order to get this one right, but it isn't terribly obvious unless you've been playing the chemistry game for a long time. So let me walk through this the *long* but expository way. It will be convenient to assume we have a total of one mole of naturally-occurring Ne atoms. [Realize that "mole" is just a number, and this statement is perfectly analogous to saying "assume we have a total of one dozen naturally-occurring Ne atoms."] 90.92% of the atoms in this sample will be ^{20}Ne, so that would be 0.9092 mol of ^{20}Ne. Similarly, we would have 0.00257 mol of ^{21}Ne and 0.0882 mol of ^{22}Ne. All together, as we assumed, that is one mole. Soooooo…

$$\%_{\text{mole}}\ \text{of}\ {}^{20}\text{Ne} = \frac{\text{moles of}\ {}^{20}\text{Ne}}{\text{total moles}} = \frac{0.9092\ \text{mol}\ {}^{20}\text{Ne}}{(\text{exactly})\,1\ \text{mol total}} = 0.9092 \times 100\% = 90.92\%$$

$$\%_{\text{mole}}\ \text{of}\ {}^{21}\text{Ne} = \frac{\text{moles of}\ {}^{21}\text{Ne}}{\text{total moles}} = \frac{0.00257\ \text{mol}\ {}^{21}\text{Ne}}{(\text{exactly})\,1\ \text{mol total}} = 0.00257 \times 100\% = 0.257\%$$

$$\%_{\text{mole}}\ \text{of}\ {}^{22}\text{Ne} = \frac{\text{moles of}\ {}^{22}\text{Ne}}{\text{total moles}} = \frac{0.0882\ \text{mol}\ {}^{22}\text{Ne}}{(\text{exactly})\,1\ \text{mol total}} = 0.0882 \times 100\% = 8.82\%$$

d. The partial pressure of an ideal gas component is directly related to the total pressure by the mole fraction of that component in the gas: Partial pressure of $A = P_A = \chi_A \cdot P_{total}$ where χ_A is the mole fraction of A in the gas phase

The mole fraction of A is exactly synonymous with the $\%_{mole}$ of A, which we have already shown to be equal to the atomic percentage $\%_{at}$ if all the components in the sample are indeed individual atoms. So all we have to do is determine what the total pressure is in the flask. That too comes from the ideal gas law, applied to the "total" gas:

$$P_{tot} V = \eta_{tot} RT \quad \Rightarrow \quad P_{tot} = \frac{\eta_{tot} RT}{V} \qquad \text{where}$$

η_{tot} = total moles of Ne atoms in the flask = (exactly) one mole (given in problem statement)

T = Temperature of the Ne in the flask = 26ºC = $299._{15}$ K (on an absolute scale)

V = The volume available to the Ne atoms in the flask = 1.00 ℓ (given in the problem statement)

R = 0.08206 $\ell \cdot atm \cdot mol^{-1} \cdot K^{-1}$ (The gas constant, from the back inside cover of your book.)

$$P_{tot} = \frac{\eta_{tot} RT}{V} = \frac{(1.00\ \text{mol})(0.08206\ \ell \cdot \text{atm} \cdot \text{mol}^{-1} \cdot \text{K}^{-1})(299._{15}\text{K})}{1.00\ \ell} = 24.5_{48}\ \text{atm}$$

So the partial pressure attributable to each Ne isotope in a flask with a total Ne pressure of 24.5 atmospheres would simply be

$$\boxed{P_{^{20}\text{Ne}} = \chi_{^{20}\text{Ne}} \cdot P_{total} = 0.9092 \times 24.5_{48}\ \text{atm} = 22.3_{19}\ \text{atm} = 22.3\ \text{atm}}$$

$$\boxed{P_{^{21}\text{Ne}} = \chi_{^{21}\text{Ne}} \cdot P_{total} = 0.00257 \times 24.5_{48}\ \text{atm} = 0.0630_{88}\ \text{atm} = 0.0631\ \text{atm}}$$

$$\boxed{P_{^{22}\text{Ne}} = \chi_{^{22}\text{Ne}} \cdot P_{total} = 0.0882 \times 24.5_{48}\ \text{atm} = 2.16_{51}\ \text{atm} = 2.17\ \text{atm}}$$

If the total Ne pressure were 1.00 atm, the partial pressures in atm would be numerically equal to the mole fractions.

[Note: Better not drop that flask, or you would be in a world of hurt! It is under a LOT of pressure!]